

The $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$ as dynamically generated states from vector meson - vector meson interaction

L. S. Geng^{*}, E. Oset[†], R. Molina[†], A. Martínez Torres[†], T. Branz^{**}, F. K. Guo[‡], L. R. Dai[§] and B. X. Sun[¶]

^{*}*School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China*
[†]*Physik Department, Technische Universität München, D-85747 Garching, Germany*

[‡]*Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46071 Spain*

^{**}*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

[‡]*Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*

[§]*Department of Physics, Liaoning Normal University, Dalian, 116029, China*

[¶]*Institute of Theoretical Physics, College of Applied Sciences, Beijing University of Technology, Beijing 100124, China.*

Abstract. We report on some recent developments in understanding the nature of the low-lying mesonic resonances $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$. In particular we show that these five resonances can be dynamically generated from vector meson–vector meson interaction in a coupled-channel unitary approach, which utilizes the phenomenologically very successful hidden-gauge Lagrangians to produce the interaction kernel between two vector mesons, which is then unitarized by the Bethe-Salpeter-equation method. The data on the strong decay branching ratios, total decay widths, and radiative decay widths of these five states, and on related J/ψ decay processes can all be well described by such an approach. We also make predictions, compare them with the results of earlier studies, and highlight observables that if measured can be used to distinguish different pictures of these resonances.

Keywords: coupled-channel unitarity, vector meson–vector meson interaction, radiative decays, J/ψ decays

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INTRODUCTION

In recent years, coupled-channel unitarity has been shown to play a very important role in understanding final state interactions and the nature of certain resonances which do not fit easily into naive valence quark models, where mesons consist of a pair of quark-antiquark and baryons three quarks. By combining coupled-channel unitarity with chiral perturbation theory, the so-called unitary chiral theories (U χ PT) have proven to be a very powerful tool to describe some of such resonances, e.g., the $f_0(600)$ [1] and the $\Lambda(1405)$ [2]. The coupled-channel unitary methods can also be extended to employ interaction kernels provided by other Lagrangians, e.g., the hidden-gauge Lagrangians [3, 4, 5, 6] and Lagrangians inspired by SU(4) flavor symmetry [7, 8] or SU(6)[SU(8)] spin-flavor symmetry [9, 10].

An interesting development in this line of research is the description of the interactions

of the lowest-lying vector mesons either with themselves [3, 4] or with other hadrons, e.g., the lowest-lying octet [5] and decuplet [6] baryons. It was shown in Ref. [4] that the interaction between two vector mesons naturally generates 11 resonances, which include not only the two scalar resonances $f_0(1370)$ and $f_0(1710)$, whose nature are still hotly debated both theoretically and experimentally, but also the tensor resonances $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$, which for a long time have been firmly believed to be simple $q\bar{q}$ states. Whether such a dynamical picture of these states is correct or partially correct can only be checked by studying as extensively as possible the corresponding theoretical implications and comparing them with data and those predicted by other approaches, such as naive quark models. For this purpose we have studied not only their strong-decay branching ratios [4] but also their radiative decay widths [11]. Furthermore we have looked at the abundant J/ψ decay data where J/ψ decays into one of these five resonances with either a vector meson [12] or a photon [13].

This paper is organized as follows. In section 2, we briefly summarize the coupled-channel unitary approach, within which the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$ are dynamically generated. In Section 3, we compare their strong decay branching ratios with available data. In Section 4, we discuss the related J/ψ decays. In Section 5, we calculate the radiative decay widths and compare with data and the predictions of other approaches. A brief summary is given in Section 6.

THEORETICAL FRAMEWORK

For the sake of completeness, in this section we provide a short description of the coupled-channel unitary approach, in which the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$ are dynamically generated. For more details, we refer the readers to Ref. [4]. The starting point of the coupled-channel unitary approach is the interaction kernel. In the present case this is provided by the hidden-gauge Lagrangians [14], which are known to describe very well the interactions between vector mesons and other hadrons including the vector mesons themselves. Once a suitable interaction kernel is chosen, the next step is to unitarize it. Here one can choose from a number of different procedures, such as the Bethe-Salpeter-equation method, which we chose, the N/D method, and the inverse amplitude method (IAM). For a short discussion of the similarities and differences between these three unitarization procedures, see, e.g., Ref. [15]

The parameters of the coupled-channel unitary approach are related to the regularization of the loops appearing in the unitarization procedure, which are cutoff values in the cutoff regularization method or subtraction constants in the dimensional regularization method. In the case of a light meson (such as the π) interacting with a heavy particle (such as the nucleon and the D mesons), model-independent determination of these parameters has been claimed. On the other hand, in our present case we simply treat them as free parameters with their values constrained by the observation that if one uses the cutoff method to regularize the loop integrals, the value of the cutoff should be around 1 GeV, to be consistent with what one would expect from a phenomenological point of view.

As explained in detail in Ref. [4], this is exactly the procedure we followed. That

is to say, we first performed the calculation using a cutoff value, Λ , of about 1 GeV and allowed Λ to vary within a reasonable range, e.g., 10~20%. This way we found that 11 resonances got dynamically generated in 9 isospin-strangeness channels and some of them can be easily associated to the experimentally well-known states, such as the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$. We then adopted the dimensional regularization method to regularize the loop integrals in order to be able to perform analytic continuation of the amplitudes to the complex plane. The values of the subtraction constants were fixed in such a way that the results obtained using the cutoff method were reproduced. We have slightly tuned the values of the subtraction constants to reproduce the masses of the three tensor states. We must stress here that in our determination of the model parameters we have not fitted the branching ratios of these states and therefore they are predictions fixed completely by the dynamics built into the model.

THE (STRONG) PARTIAL DECAY WIDTHS AND BRANCHING RATIOS

A detailed description of the calculation of the partial decay widths can be found in Ref. [16]. In Table 1, we tabulate the (strong) branching ratios of the states $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, $K_2^*(1430)$ in comparison with available data. It is clear that our results for the two f_2 states agree very well with the data. For the $f_0(1370)$, according to the PDG [17], the $\rho\rho$ mode is dominant. In our approach, however, the $\pi\pi$ mode is dominant, which is consistent with the results of Ref. [18] and the recent analysis of D. V. Bugg [19]. For the $f_0(1710)$, using the branching ratios given in Table 1, we obtained $\Gamma(\pi\pi)/\Gamma(K\bar{K}) < 1\%$ and $\Gamma(\eta\eta)/\Gamma(K\bar{K}) \sim 49\%$. On the other hand, the PDG gives the following averages: $\Gamma(\pi\pi)/\Gamma(K\bar{K}) = 0.41^{+0.11}_{-0.17}$, and $\Gamma(\eta\eta)/\Gamma(K\bar{K}) = 0.48 \pm 0.15$ [17]. Our calculated branching ratio for the $\eta\eta$ channel is in agreement with their average, while the ratio for the $\pi\pi$ channel is much smaller. However, we notice that the above PDG $\Gamma(\pi\pi)/\Gamma(K\bar{K})$ ratio is taken from the BES data on $J/\psi \rightarrow \gamma\pi^+\pi^-$ [20], which comes from a partial wave analysis that includes seven resonances. On the other hand, the BES data on $J/\psi \rightarrow \omega K^+K^-$ [21] give an upper limit $\Gamma(\pi\pi)/\Gamma(K\bar{K}) < 11\%$ at the 95% confidence level. Clearly more analysis is needed to settle the issue.

In Table 1, one can see that the dominant decay mode of the $K_2^*(1430)$ is $K\pi$ both theoretically and experimentally. However, other modes, such as ρK , $K^*\pi$, and $K^*\pi\pi$, account for half of its decay width according to the PDG [17]. This is consistent with the fact that our $K_2^*(1430)$ is narrower than its experimental counterpart [4].

RELATED J/ψ DECAYS

J/ψ decays into a vector meson ρ^0 , ω , ϕ and one of the tensor mesons $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$.

J/ψ decays offer a good opportunity to test the dynamical picture of the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$. One of such processes is J/ψ decay into

TABLE 1. Branching ratios of the $f_0(1710)$, $f_0(1370)$, $f_2(1270)$, $f_2'(1525)$, and $K_2^*(1430)$ in comparison with data [17].

	$\Gamma(\pi\pi)/\Gamma(\text{total})$		$\Gamma(\eta\eta)/\Gamma(\text{total})$		$\Gamma(K\bar{K})/\Gamma(\text{total})$		$\Gamma(VV)/\Gamma(\text{total})$	
	Our model	Data	Our model	Data	Our model	Data	Our model	Data
$f_0(1370)$	$\sim 72\%$		$< 1\%$		$\sim 10\%$		$\sim 18\%$	
$f_0(1710)$	$< 1\%$		$\sim 27\%$		$\sim 55\%$		$\sim 18\%$	
$f_2(1270)$	$\sim 88\%$	84.8%	$< 1\%$	$< 1\%$	$\sim 10\%$	4.6%	$< 1\%$	
$f_2'(1525)$	$< 1\%$	0.8%	$\sim 21\%$	10.4%	$\sim 66\%$	88.7%	$\sim 13\%$	
$K_2^*(1430)$	$\sim 93\%$	49.9%	$\sim 5\%$	$< 1\%$	$\sim 2\%$			

a vector meson ρ^0 , ω , or ϕ and one of the tensor mesons $f_2(1270)$, $f_2'(1525)$, $K_2^*(1430)$. Furthermore, if we are only interested in the ratios of the partial decay widths we could ignore absolute normalization and greatly simplify the calculation by noting that J/ψ is a SU(3) singlet and the tensor resonances are dynamically generated from vector meson–vector meson interactions. More details can be found in Ref. [12].

To study the ratios of different decay rates, e.g.,

$$R_1 \equiv \frac{\Gamma_{J/\psi \rightarrow \phi f_2(1270)}}{\Gamma_{J/\psi \rightarrow \phi f_2'(1525)}}, \quad R_2 \equiv \frac{\Gamma_{J/\psi \rightarrow \omega f_2(1270)}}{\Gamma_{J/\psi \rightarrow \omega f_2'(1525)}}, \quad (1)$$

$$R_3 \equiv \frac{\Gamma_{J/\psi \rightarrow \omega f_2(1270)}}{\Gamma_{J/\psi \rightarrow \phi f_2(1270)}}, \quad R_4 \equiv \frac{\Gamma_{J/\psi \rightarrow K^{*0} \bar{K}_2^{*0}(1430)}}{\Gamma_{J/\psi \rightarrow \omega f_2(1270)}}, \quad (2)$$

we needed only one parameter v , which can be fixed by fitting our predictions to data. Upon minimization of the χ^2 function we obtained an optimal solution: $v = 1.45$. For our estimate of the theoretical uncertainties, see Ref. [12]. As can be seen from Table 2, the overall agreement of our results with the data is reasonable. We obtained four independent ratios with just one parameter. On the other hand this parameter can be

TABLE 2. Comparison between the experimental and the theoretical results.

	Experiment	Theory
R_1	0.22 - 0.47 ($0.33^{+0.14}_{-0.11}$)	0.13 - 0.61 ($0.28^{+0.33}_{-0.15}$)
R_2	12.33 - 49.00 ($21.50^{+27.50}_{-9.17}$)	2.92 - 13.58 ($5.88^{+7.70}_{-2.96}$)
R_3	11.21 - 23.08 ($15.85^{+7.23}_{-4.65}$)	6.18 - 19.15 ($10.63^{+8.52}_{-4.45}$)
R_4	0.55 - 0.89 ($0.70^{+0.19}_{-0.15}$)	0.83 - 2.10 ($1.33^{+0.77}_{-0.50}$)

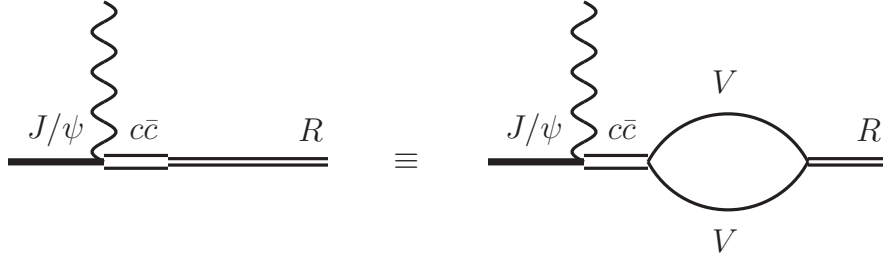


FIGURE 1. Schematic representation of J/ψ decay into a photon and one dynamically generated resonance.

related to λ_ϕ of Ref. [22] as done in Ref. [23] through

$$\lambda_\phi = \sqrt{2} \left(\frac{v-1}{v+2} \right) \quad (3)$$

which provides a value of $\lambda_\phi = 0.18$, very close to the one obtained in Refs. [22, 23], $\lambda_\phi = 0.13 - 0.20$. Although we have different physics than in Ref. [22, 23] since we have the production of pairs of vector mesons rather than pseudoscalar mesons, and we have also tensor states rather than scalars, it is gratifying to see that the value of λ_ϕ , which is a measure of the subdominant, double OZI suppressed, mechanism in $J/\psi \rightarrow \phi V' V'$ (see Fig. 1b of Ref. [12]), is a small number, comparable in size and sign to the one obtained in Refs. [22, 23].

The success in the description of the experimental data is by no means trivial and can be traced back to the particular couplings of the resonance to the $V' V'$ states. Note that the important couplings to $\rho\rho$ and $K^* \bar{K}^*$ have the same relative sign for the $f_2(1270)$ and opposite relative sign for the $f_2'(1525)$ (see Table I of Ref. [4]). This feature is essential to the success of the results. Should all the couplings have the same sign it would have been impossible to get any reasonable fit to the data (see Fig. 7 of Ref. [12]).

J/ψ decays into a photon and one of the $f_2(1270)$, $f_2'(1525)$, $f_0(1370)$, and $f_0(1710)$

In a similar way to what was shown above, one can also study the J/ψ decays into a photon and one of the $f_2(1270)$, $f_2'(1525)$, $f_0(1370)$, and $f_0(1710)$ [13]. Such a process is schematically shown in Fig.1. To remove the dependence of the $c\bar{c}VV$ vertex on spin we constructed the following two ratios

$$R_T = \Gamma_{J/\psi \rightarrow \gamma f_2(1270)} / \Gamma_{J/\psi \rightarrow \gamma f_2'(1525)}, \quad (4)$$

$$R_S = \Gamma_{J/\psi \rightarrow \gamma f_0(1370)} / \Gamma_{J/\psi \rightarrow \gamma f_0(1710)}. \quad (5)$$

The results are compared with available data in Table 3 with the theoretical uncertainties estimated in Ref. [13]. Our result for R_T is in reasonable agreement with data while there

TABLE 3. Ratios of $R_T = \Gamma_{J/\psi \rightarrow \gamma f_2(1270)}/\Gamma_{J/\psi \rightarrow \gamma f'_2(1525)}$ and $R_S = \Gamma_{J/\psi \rightarrow \gamma f_0(1370)}/\Gamma_{J/\psi \rightarrow \gamma f_0(1710)}$ within the molecular model and the quark model in comparison with data [24].

	Molecular picture	Quark model	Data
R_T	2 ± 1	2.2	$3.18^{+0.58}_{-0.64}$
R_S	1.2 ± 0.3	2.2 – 2.5	
R_S/R_T	0.6 ± 0.1	1 – 1.1	

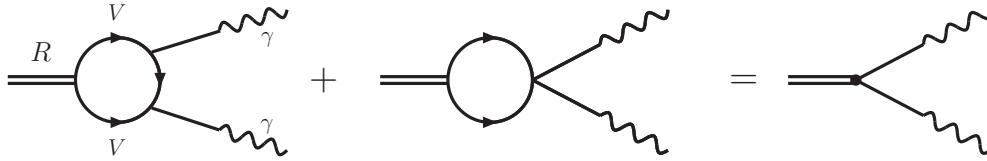


FIGURE 2. Two-photon decay of a dynamically generated resonance from vector meson-vector meson interaction.

is no data for R_S . To show the differences between our dynamical picture and the naive quark model where the $f_0(1370)$ and $f_0(1710)$ belong to the same scalar nonet while the $f_2(1270)$ and $f'_2(1525)$ belong to the same tensor nonet, we have calculated R_T and R_S using a rather naive quark model. The results obtained are also shown in Table 3. It is quite amazing to notice that although the quark model yields a R_T consistent with our prediction, it gives a quite different R_S than ours. Experimental measurement of R_S will be very useful to distinguish between these two different pictures of the $f_0(1370)$ and $f_0(1710)$.

RADIATIVE DECAYS OF THE $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, AND $K_2^*(1430)$

Radiative decay provides a very clean probe of the structure of hadronic states. For instance, the non-observation of the $f_0(1500)$ decaying into two photons has been used to support its dominant glue nature [25]. In Ref. [11], we have calculated the two-photon ($\gamma\gamma$) and the vector-meson-photon ($V\gamma$) decay widths of the 11 dynamically generated states from vector meson–vector meson interactions. Because these 11 states are built from vector meson – vector meson interactions and because in the hidden-gauge Lagrangians photons do not couple directly to charged vector mesons but must first convert into neutral vector mesons, the calculation of the radiative decay widths is quite straightforward. As shown in Fig. 2, one simply needs the couplings of the resonance of interest to the vector meson – vector meson coupled channels. This has been done in

TABLE 4. Radiative decay widths of the $f'_2(1525)$ obtained in the present work in comparison with those obtained in the covariant oscillator quark model (COQM) [26].

	COQM [26]	Present work
$f'_2(1525) \rightarrow \gamma\gamma$		0.05
$f'_2(1525) \rightarrow \rho^0\gamma$	4.8	72
$f'_2(1525) \rightarrow \omega\gamma$	0	224
$f'_2(1525) \rightarrow \phi\gamma$	104	286

Ref. [4]. With these couplings, one can easily obtain the $\gamma\gamma$ and $V\gamma$ decay widths as

$$\Gamma_{\gamma\gamma} = \frac{1}{2S+1} \frac{1}{16\pi M_R} \frac{1}{2} \times \sum_{\text{polarization}} |T_{\gamma\gamma}^{(R)}|^2, \quad (6)$$

$$\Gamma_{V\gamma} = \frac{1}{2S+1} \frac{1}{8\pi M_R} \frac{|p_\gamma|}{M_R} \times \sum_{\text{polarization}} |T_{V\gamma}^{(R)}|^2, \quad (7)$$

where M_R is the resonance mass, p_γ is the photon momentum in the rest frame of the resonance R , S is the resonance spin, and $T_{\gamma\gamma}^{(R)}$ ($T_{V\gamma}^{(R)}$) is defined in Ref. [11].

Our predictions for the VV and $V\gamma$ decay widths of the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, and $K_2^*(1430)$ are tabulated in Tables I-VIII of Ref. [11]. Our results agree reasonably well with the data, but show distinct behavior compared to the predictions of other approaches. For instance, the $f'_2(1525) \rightarrow \rho^0\gamma$ and $f'_2(1525) \rightarrow \omega\gamma$ partial decay widths (see Table 4) are quite different from those predicted by the covariant oscillator quark model (COQM). An experimental measurement of these two decay modes, particularly their ratio, should be very useful to distinguish the two different pictures of the $f'_2(1525)$. Similar distinct results have been observed for the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, and $f'_2(1525)$ [11].

SUMMARY

The nature of a hadronic state is definitely more complex than what one used to assume. In a naive quark model, baryons consist of three quarks (qqq) and mesons a pair of quark and anti-quark ($q\bar{q}$). In recent years, studies have found evidences that baryons and mesons often contain multi-quark components, which in certain cases may even be more important than the qqq and $q\bar{q}$ structures. In the u , d , s flavor sector, the $f_0(600)$ and the $\Lambda(1405)$ are believed to be such states. The newly experimentally measured X , Y , Z particles are examples in the charmonium sector.

Coupled-channel unitary approaches are particularly suitable for describing s -wave resonances whose wave function contains large meson-meson or meson-baryon compo-

nents. In the past few years, many resonances have been generated in coupled-channel unitary approaches and they often are referred to as "dynamically generated resonances." However, generating resonances is only the first step. One has to test as extensively as possible the consequences of such descriptions. In this talk, we have reported on a series of recent works exploring the possible dynamical picture of the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f_2'(1525)$, and $K_2^*(1430)$. Our studies have shown that all existing data are consistent with the dynamical picture and therefore the wave function of these five states may contain important multi-quark components in the form of vector meson-vector meson configuration. Furthermore, for certain observables our studies have shown that this dynamical picture predicts distinct and (in principle) detectable patterns compared to, e.g., various quark models. Experimental verification of these predictions will be very useful to distinguish between different pictures of the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f_2'(1525)$, and $K_2^*(1430)$.

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REFERENCES

1. J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid. A **652**, 407 (1999)]; N. Kaiser, Eur. Phys. J. A **3**, 307 (1998); V. E. Markushin, Eur. Phys. J. A **8**, 389 (2000); A. Dobado and J. R. Pelaez, Phys. Rev. D **56**, 3057 (1997); J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999) [Erratum-ibid. D **60**, 099906 (1999); Erratum-ibid. D75, 099903 (2007)].
2. N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A **594**, 325 (1995); E. Oset and A. Ramos, Nucl. Phys. A **635**, 99 (1998); J. A. Oller and U. G. Meissner, Phys. Lett. B **500**, 263 (2001); C. Garcia-Recio, M. F. M. Lutz and J. Nieves, Phys. Lett. B **582**, 49 (2004); D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A **725**, 181 (2003); C. Garcia-Recio, J. Nieves and L. L. Salcedo, Phys. Rev. D **74**, 034025 (2006); T. Hyodo, S. I. Nam, D. Jido and A. Hosaka, Phys. Rev. C **68**, 018201 (2003).
3. R. Molina, D. Nicmorus and E. Oset, Phys. Rev. D **78**, 114018 (2008).
4. L. S. Geng and E. Oset, Phys. Rev. D **79**, 074009 (2009).
5. E. Oset and A. Ramos, Eur. Phys. J. A **44**, 445 (2010).
6. S. Sarkar, B. X. Sun, E. Oset and M. J. V. Vacas, Eur. Phys. J. A **44**, 431 (2010).
7. R. Molina, H. Nagahiro, A. Hosaka and E. Oset, Phys. Rev. D **80**, 014025 (2009).
8. D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).
9. C. Garcia-Recio, V. K. Magas, T. Mizutani, J. Nieves, A. Ramos, L. L. Salcedo and L. Tolos, Phys. Rev. D **79**, 054004 (2009).
10. C. Garcia-Recio, L. S. Geng, J. Nieves and L. L. Salcedo, arXiv:1005.0956 [hep-ph].
11. T. Branz, L. S. Geng and E. Oset, Phys. Rev. D **81**, 054037 (2010).
12. A. Martinez Torres, L. S. Geng, L. R. Dai, B. X. Sun, E. Oset and B. S. Zou, Phys. Lett. B **680**, 310 (2009).
13. L. S. Geng, F. K. Guo, C. Hanhart, R. Molina, E. Oset and B. S. Zou, Eur. Phys. J. A **44**, 305 (2010).
14. M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985); M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. **164**, 217 (1988).
15. L. S. Geng, E. Oset, J. R. Pelaez and L. Roca, Eur. Phys. J. A **39**, 81 (2009).
16. L. S. Geng, E. Oset, R. Molina and D. Nicmorus, PoS E **FT09**, 040 (2009).
17. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
18. M. Albaladejo and J. A. Oller, Phys. Rev. Lett. **101**, 252002 (2008).

19. D. V. Bugg, Eur. Phys. J. C **52**, 55 (2007).
20. M. Ablikim *et al.*, Phys. Lett. B **642**, 441 (2006).
21. M. Ablikim *et al.* [BES Collaboration], Phys. Lett. B **603** (2004) 138.
22. U. G. Meissner and J. A. Oller, Nucl. Phys. A **679**, 671 (2001).
23. L. Roca, J. E. Palomar, E. Oset and H. C. Chiang, Nucl. Phys. A **744**, 127 (2004).
24. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
25. C. Amsler, Phys. Lett. B **541**, 22 (2002).
26. S. Ishida, K. Yamada and M. Oda, Phys. Rev. D **40**, 1497 (1989).

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